1.3 The Rigid Body

A body is called a *rigid body* if it does not deform under the influence of forces; the distances between different points of the body remain constant. This is, of course, an idealization of a real body composed of a reasonably stiff material which in many cases is fulfilled in a good approximation. From experience with such bodies it is known that a single force may be applied at any point on the line of action without changing its effect on the body as a whole (*principle of transmissibility*).

This principle is illustrated in Fig. 1.4. In the case of a deformable sphere, the effect of the force depends on the point of

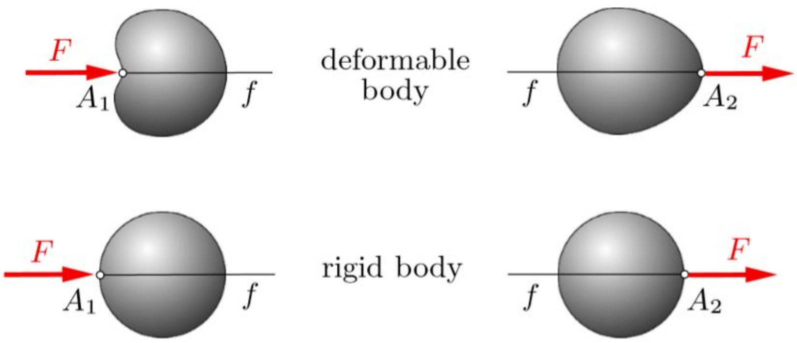

Fig. 1.4

application. In contrast, for a rigid sphere the effect of the force F on the entire body is the same, regardless of whether the body is pulled or pushed. In other words:

> The effect of a force on a rigid body is independent of the location of the point of application on the line of action. The forces acting on rigid bodies are "sliding vectors": they can arbitrarily be moved along their action lines.

A parallel displacement of forces changes their effect considerably. As experience shows, a body with weight W can be held in equilibrium if it is supported appropriately (underneath the center of gravity) by the force F, where $F = W$ (Fig. 1.5a). Displacing force F in a parallel manner causes the body to rotate (Fig. 1.5b).

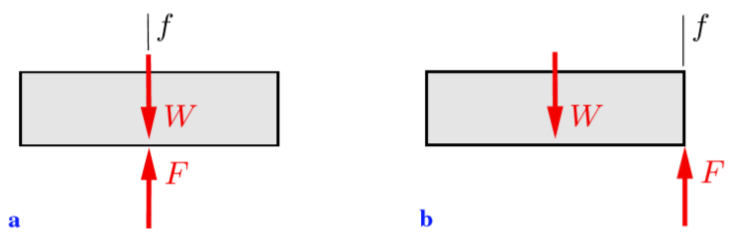

Fig. 1.5

In short:

The forces that two bodies exert upon each other are of the same magnitude but of opposite directions and they lie on the same line of action.

This principle, which Newton succinctly expressed in Latin as

actio = reactio

is the third of Newton's axioms (cf. Volume 3). It is valid for long-range forces as well as for short-range forces, and it is independent of whether the bodies are at rest or in motion.

2.1 Addition of Forces in a Plane

Consider a body that is subjected to two forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$, whose lines of action intersect at point A (Fig. 2.1a). It is postulated that the two forces can be replaced by a statically equivalent force $\boldsymbol{R}$. This postulate is an axiom; it is known as the *parallelogram law of forces*. The force $\boldsymbol{R}$ is called the *resultant* of $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$. It is the diagonal of the parallelogram for which $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are adjacent sides. The axiom may be expressed in the following way:

The effect of two nonparallel forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ acting at a point A of a body is the same as the effect of the single force $\boldsymbol{R}$ acting at the same point and obtained as the diagonal of the parallelogram formed by $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$.

The construction of the parallelogram is the geometrical representation of the summation of the vectors (see Appendix A.1):

$$\boldsymbol{R} = \boldsymbol{F}_1 + \boldsymbol{F}_2 \,. \tag{2.1}$$

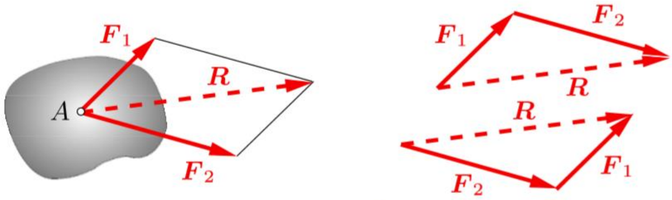

Fig. 2.1 a b

We now investigate the conditions under which a body is in *equilibrium* when subjected to the action of forces. It is known from experience that a body that was originally at rest stays at rest if two forces of equal magnitude are applied that have the same line of action and are oppositely directed (Fig. 2.9). In other words:

Two forces are in equilibrium if they are oppositely directed on the same line of action and have the same magnitude.

This means that the sum of the two forces, i.e., their resultant, has to be the zero vector:

$$\boldsymbol{R} = \boldsymbol{F}_1 + \boldsymbol{F}_2 = \boldsymbol{0}\,. \tag{2.9}$$

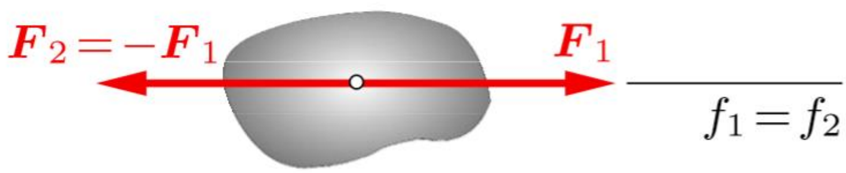

Fig. 2.9